Lösungen vom 18.3.20

S. 168, Nr. 7

(1) $n=4$: $f(x)=ax^4+bx^3+cx^2+dx+e$

(2) achsensymm.: $f(x)=ax^4+cx^2+e$; $f'(x)=4ax^3+2cx$; $f''(x)=12ax^2+2c$

(3) $f(0)=-1$: $a\cdot 0^4+c\cdot 0^2+e=-1$

(4) $f(1)=-3$: $a\cdot 1^4+c\cdot 1^2+e=-3$

(5) $f'(1)=0$: $4\cdot a\cdot 1^3+2c\cdot 1=0$

$$\Rightarrow \left|\begin{array}{l} e=-1 \\ a+c+e=-3 \\ 4a+2c=0 \end{array}\right| \begin{array}{l} \\ \cdot 2 \\ - \end{array} \Rightarrow \left|\begin{array}{l} e=-1 \\ -2a+2e=-6 \\ 4a+2c=0 \end{array}\right|$$

$e=-1 \Rightarrow$ in II: $-2a-2=-6 \Rightarrow a=2$

in III: $4\cdot 2+2c=0 \Rightarrow c=-4$

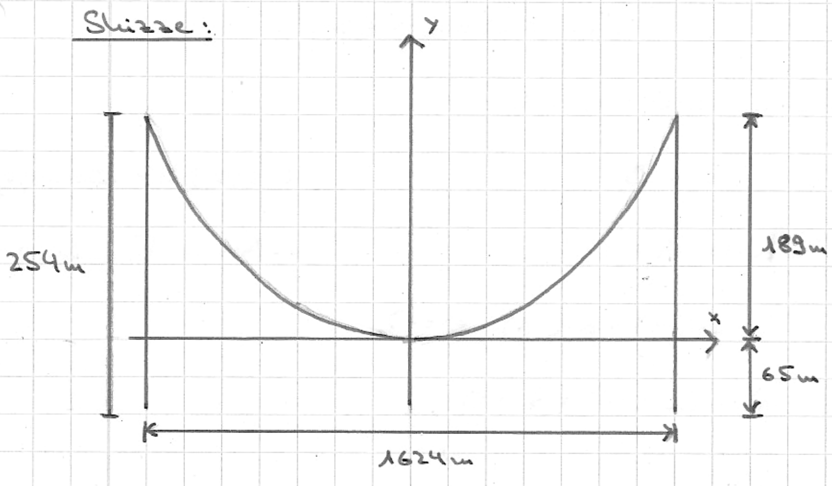
Überprüfung (↔ Ist $(1|-3)$ wirklich ein Tiefpunkt?): $f''(1)=12\cdot 2\cdot 1+2\cdot(-4)=16 > 0$

$\Rightarrow$ TP

$$\Rightarrow \underline{\underline{f(x)=2x^4-4x^2-1}}$$

S. 168, Nr. 10

Skizze:



(1) $n=2$: $f(x)=ax^2+bx+c$

(2) achsensymm.: $f(x)=ax^2+c$

(3) $f(0)=0$: $a\cdot 0^2+c=0$

(4) $f(812)=189$: $a\cdot 812^2+c=189$

$$\Rightarrow \left|\begin{array}{l} c=0 \\ 812^2a+c=189 \end{array}\right| \Rightarrow a=189:812^2\approx 2{,}866\cdot 10^{-4}$$

$$\Rightarrow \underline{\underline{f(x)=2{,}866\cdot 10^{-4}x^2}}$$

(Die Verschiebung hängt von eurem KS ab.)

S. 169, Nr. 13

a) (1) $n=4: f(x) = ax^4 + bx^3 + cx^2 + dx + e$

(2) achsensymm.: $f(x) = ax^4 + cx^2 + e \; ; \; f'(x) = 4ax^3 + 2cx$

(3) $f(0) = 2: \quad a \cdot 0^4 + c \cdot 0^2 + e = 2$

(4) $f(1) = 0: \quad a \cdot 1^4 + c \cdot 1^2 + e = 0$

(5) $f'(1) = 0: \quad 4 \cdot a \cdot 1^3 + 2c \cdot 1 = 0$

$$\Rightarrow \left| \begin{array}{rcl} & e & = 2 \\ a + c & + e & = 0 \\ 4a + 2c & & = 0 \end{array} \right| \quad \Rightarrow \text{Lös. auf S. 367}$$

b) (1) $n=4: f(x) = ax^4 + bx^3 + cx^2 + dx + e$

(2) achsensymm.: $f(x) = ax^4 + \cancel{b}x^2 + e \; ; \; f'(x) = 4ax^3 + 2cx \; ; \; f''(x) = 12ax^2 + 2c$

(3) $f(1) = 0: \quad a \cdot 1^4 + c \cdot 1^2 + e = 0$

(4) $f''(1) = 0: \quad 12a \cdot 1^2 + 2c = 0$

(5) $f'(1) = 8: \quad 4a \cdot 1^3 + 2c \cdot 1 = 8$

$$\Rightarrow \left| \begin{array}{rcl} a + c + e & = 0 \\ 12a + 2c & = 0 \\ 4a + 2c & = 8 \end{array} \right|$$

S. 169, Nr. 14

a) (1) $n=4: f(x) = ax^4 + bx^3 + cx^2 + dx + e \; ; \; f'(x) = 4ax^3 + 3bx^2 + 2cx + d$

$f''(x) = 12ax^2 + 6bx + 2c$

(2) $f(0) = 0: \quad a \cdot 0^4 + b \cdot 0^3 + c \cdot 0^2 + d \cdot 0 + e = 0$

(3) $f''(0) = 0: \quad 12a \cdot 0^2 + 6b \cdot 0 + 2c = 0$ (W(0|0) ist Wendepunkt)

(4) $f'(0) = 0: \quad 4a \cdot 0^3 + 3b \cdot 0^2 + 2c \cdot 0 + d = 0$ (x-Achse Tangente)

(5) $f(-1) = -2: \quad a \cdot (-1)^4 + b \cdot (-1)^3 + c \cdot (-1)^2 + d \cdot (-1) + e = -2$

(6) $f'(-1) = 0: \quad 4a \cdot (-1)^3 + 3b \cdot (-1)^2 + 2c \cdot (-1) + d = 0$

$$\Rightarrow \left| \begin{array}{rcl} e & = 0 \\ 2c & = 0 \\ d & = 0 \\ a - b + c - d + e & = -2 \\ -4a + 3b - 2c + d & = 0 \end{array} \right|$$

S. 169, 14

$$f''(x) = 12ax^2 + 6bx + 2c$$

b) (1) $n = 4$: $f(x) = ax^4 + bx^3 + cx^2 + dx + e$; $f'(x) = 4ax^3 + 3bx^2 + 2cx + d$

(2) $f(0) = 0$: $a \cdot 0^4 + b \cdot 0^3 + c \cdot 0^2 + d \cdot 0 + e = 0$

(3) $f'(0) = 0$: $4a \cdot 0^3 + 3b \cdot 0^2 + 2c \cdot 0 + d = 0$

(4) $f(-2) = 2$: $a \cdot (-2)^4 + b \cdot (-2)^3 + c \cdot (-2)^2 + d \cdot (-2) + e = 2$

(5) $f'(-2) = 0$: $4a \cdot (-2)^3 + 3b \cdot (-2)^2 + 2c \cdot (-2) + d = 0$

(6) $f''(-2) = 0$: $12a \cdot (-2)^2 + 6b \cdot (-2) + 2c = 0$

$$\Rightarrow \left| \begin{array}{l} e = 0 \\ d = 0 \\ 16a - 8b + 4c - 2d + e = 2 \\ -32a + 12b - 4c + d = 0 \\ 48a - 12b + 2c = 0 \end{array} \right|$$

S. 169, 15

(1) $n = 3$: $f(x) = ax^3 + bx^2 + cx + d$; $f'(x) = 3ax^2 + 2bx + c$; $f''(x) = 6ax + 2b$

(2) $f(2) = 0$: $a \cdot 2^3 + b \cdot 2^2 + c \cdot 2 + d = 0$

(3) $f''(2) = 0$: $6a \cdot 2 + 2b = 0$

(4) $f'(3) = 0$: $3a \cdot 3^2 + 2 \cdot b \cdot 3 + c = 0$

$$\Rightarrow \left| \begin{array}{l} 8a + 4b + 2c + d = 0 \\ 12a + 2b = 0 \\ 27a + 6b + c = 0 \end{array} \right| \Rightarrow a \overset{!}{=} t \Rightarrow 2b = -12t \Rightarrow b = -6t$$

in III : $27t + 6 \cdot (-6t) + c = 0 \Rightarrow 9t + c = 0 \Rightarrow c = -9t$

in I : $8t + 4 \cdot (-6t) + 2 \cdot (-9t) + d = 0 \Rightarrow 2t + d = 0 \Rightarrow d = -2t$

$\Rightarrow f(x) = tx^3 + 6tx^2 - 9tx - 2t = t(x^3 - 6x^2 - 9x - 2)$

$\Rightarrow f'(x) = t \cdot (3x^2 - 12x - 9)$; $f''(x) = t \cdot (6x - 12)$; $f'''(x) = 6t$

Bedingung: HP bei $x = 3 \Rightarrow f'(3) \overset{!}{<} 0 \Leftrightarrow \cancel{t \cdot (3 - 12 - 9) = t \cdot (-18) < 0}$

$\Leftrightarrow t \cdot (27 - 36 - 9) = -18t \overset{!}{<} 0 \Leftrightarrow t < 0 \Rightarrow f'''(x) = 6t \neq 0$ für $t < 0$

$\Rightarrow$ Für $t < 0$ existiert die gesuchte Funktion.